Exercise 1: The Distortion Energy Criterion of yielding assumes that yielding starts when the distortion energy at a point in a solid becomes equal to the distortion energy at yield in simple tension of the same material.

(1) Show that the energy of distortion per unit volume can be expressed as,

$$W_d^*(\sigma_{ij}) = \frac{1+v}{6E}\left[\left(\sigma_{11}-\sigma_{22}\right)^2+\left(\sigma_{22}-\sigma_{33}\right)^2+\left(\sigma_{33}-\sigma_{11}\right)^2+6\left(\sigma_{12}^2+\sigma_{23}^2+\sigma_{31}^2\right)\right] \qquad \text{(a)}$$

(2) If the yield stress of the material in uniaxial tension is σ_Y show that this criterion is expressed as,

$$\sigma_Y = \frac{1}{\sqrt{2}}\left[\left(\sigma_{11}-\sigma_{22}\right)^2+\left(\sigma_{22}-\sigma_{33}\right)^2+\left(\sigma_{33}-\sigma_{11}\right)^2+6\left(\sigma_{12}^2+\sigma_{23}^2+\sigma_{31}^2\right)\right]^{1/2} \qquad \text{(b)}$$

Exercise 2: The stress state at a point of a solid is,

$$[\sigma] = \begin{pmatrix} \sigma & \tau & 0 \\ \tau & \sigma & 0 \\ 0 & 0 & \sigma \end{pmatrix}$$

where σ, τ are given stress. What is the yield condition according to (a) Tresca and (b) V Mises criteria?

Exercise 3: A thick-walled cylinder, with open ends, internal radius r_i and external radius $2\,r_i$ is subjected to internal pressure P_i. The tensile yield stress of the material is σ_Y. Determine P_i at the onset of yielding using the Tresca and V Mises yield criteria. Calculate the displacement at the onset of yielding at the internal surface of the cylinder (modulus of elasticity and Poisson ratio E, v are known).

Exercise 4: Express the plastic strain increment ratios (C.24a, Appendix C) for

(1) Simple tension $\sigma_{11} = \sigma_Y$

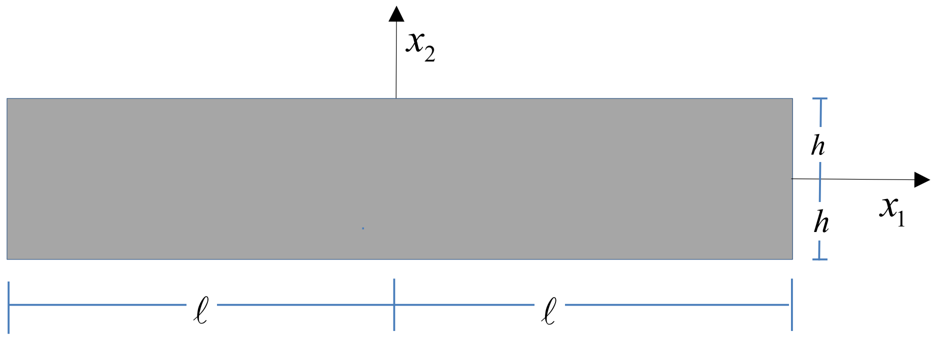
(2) Biaxial stress with $\sigma_{11} = -\sigma_Y/\sqrt{3},\ \sigma_{22} = \sigma_Y/\sqrt{3},\ \sigma_{33} = \sigma_{12} = \sigma_{23} = \sigma_{13} = 0$

(3) Pure shear $\sigma_{12} = \sigma_Y$

Problems from a previous examination

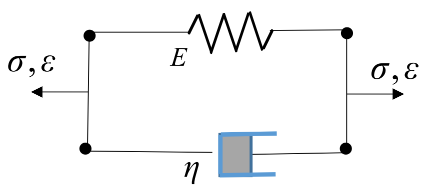
Problem A: A rectangular thin beam, shown in the Figure, with thickness t, length $2l$ and height $2h$ is subjected to a temperature variation though its height $T(x_2)$. The beam is free of surface and body forces.

1. Express the stress and strain distributions across the height.
2. Formulate clearly the boundary conditions.



Problem B:

1: derive the constitutive equation for the Kelvin-Voight model shown in the Figure below,



2: derive the equation of state for the following three-parameter model shown below,